

TRANSIENT EFFECT OF A BOGER FLUID FLOWING AROUND A CONFINED CYLINDER*

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Abstract This paper is concerned with the numerical simulation of the transient effect of an inertialess Boger flow past a confined circular cylinder and the comparison of predictions with particle image velocimetry (PIV) measurements given by Shiang *et al.*. Dynamic simulation based on the Oldroyd-B constitutive model was carried out using a Lagrangian-Eulerian algorithm. The evolution of velocity field was obtained for the flow at two Deborah (De) numbers, i.e. $De = 1.2$ and 3.0 . At low De , the flow reached steady state rapidly, and showed a symmetric flow regime. However, at high De , the time required to reach steady flow behind the cylinder increased significantly, and the distribution of the velocity field appears to be asymmetric with respect to the stagnation line. Fairly good agreement between the numerical results and the experimental observations is reported. It can be concluded that both the experimental measurements and the present simulations indicate that the elasticity of the polymeric flow strongly affect the flow regime of viscoelastic flow around a confined cylinder.

Keywords Transient effect, Flow around a cylinder, Lagrangian-Eulerian algorithm, Oldroyd-B model

INTRODUCTION

The fundamental research work on viscoelastic flow is often complicated by the intrinsic complexity of the flow, which includes the effects related to elasticity, shear thinning and fluid inertia. Boger^[1] pioneered experiments in which these effects were isolated and controlled. In recent years, one class of constant-viscosity elastic liquids (Boger fluids) has been extensively used to study the dynamics of viscoelastic flows^[2]. These liquids have large elasticity, large and nearly constant viscosity for minimization of the inertial and shear thinning effects. Therefore, the investigations on complex flows of such fluid have been simplified to an extent that the elastic effects can be directly observed by experiments or predicted by a relatively simple constitutive equation like the Oldroyd-B model within a suitable range of extension rates.

In recent experimental work of Shiang *et al.*^[3], the authors used particle image velocimetry (PIV) to measure the full-field instantaneous velocity in a PIB-based (polyisobutylene) Boger fluid flowing around a confined cylinder. They examined the effect of both a rheological parameter (elasticity) and a geometric parameter (eccentricity) on the flow regime at Deborah numbers (De) ranging from 0.6 to 3.0. Our current interest is limited to the first part of their results, where the transient effect caused by elasticity was systematically investigated. For viscoelastic flow past a centered cylinder, Shiang *et al.* examined the evolution of the velocity fields in the downstream wake region of the cylinder. They concluded that, for low flow rates (e.g. $De = 0.6$ and 1.2), the flow reached steady flow conditions rapidly. But as the flow rate was increased (e.g. $De = 1.8$, 2.4 and 3.0), the time required to reach steady flow behind the cylinder increased significantly. For $De = 3.0$, the flow regime appeared to be obviously asymmetric with respect to the stagnation line.

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The aim of this work includes two aspects. First, we used a mixed Lagrangian-Eulerian algorithm and Oldroyd-B constitutive model to simulate the transient effect (caused by elasticity) of the flows of PIB-based Boger fluid at $De = 1.2$ and 3.0 , respectively. Then, we compared the numerical results with the experimental measurements of Shiang *et al.*^[3] in order to ensure whether the algorithm is physically correct, and to what extent it can reproduce experimental observations.

In section 2, we define problems relevant to this work, present the Oldroyd-B constitutive model and select material parameters for it. In section 3, the numerical results of velocity are reported and compared with the experimental observations of Shiang *et al.*. Finally, in section 4 we end it with a summary.

PROBLEM DEFINITION

Lagrangian-Eulerian Scheme

The Lagrangian-Eulerian method used in the present work was developed by Yuan *et al.* in 1993^[4], and recently improved by Yuan and Doi^[5]. As a stress-splitting scheme, it solves the history-dependent quantities locally in a co-moving grid. The solution space is subdivided by dual grids: the Delauney triangle and the Voronoi polygon. The former determines the connection of fluid elements, and the latter describes the conformation of fluid elements. During the dynamic process, the configuration of each cell changes continuously according to the Voronoi rule. Therefore, the calculation of the deformation history of every element is straightforward, and the viscoelastic extra-stress can be obtained according to the specific constitutive equation. At every physical time step, velocity and pressure fields are accurately solved by iterating the conservation equations, which are strictly satisfied within each Voronoi cell. Details of mesh connection and finite-difference scheme can be seen elsewhere^[4, 5].

Geometry

A schematic diagram of the geometry associated with the flow around a cylinder in a tank is given in Fig. 1. The ratio of the tank half width to the cylinder radius R is 16, and the ratio of the tank length to R is 120. In each simulation, the cylinder is positioned at the origin of the Cartesian coordinate system, and the tank moves with constant velocity along the axial direction. The total distance that the tank moves relative to the cylinder is 80R. It can be seen from the numerical results that the elastic effect can be clearly observed within such a distance of motion.

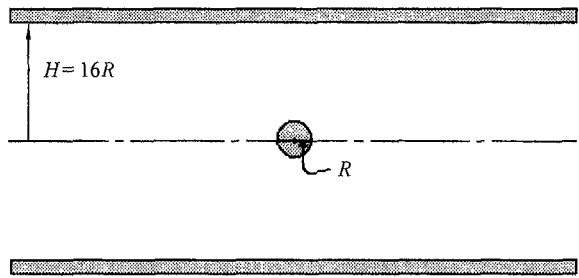


Fig. 1 The geometry associated with the flow around a confined cylinder problem

Basic Equations

For an incompressible fluid, the system of mass and momentum conservation equations describing the fluid motion can be written as:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \nabla \cdot \boldsymbol{\sigma}(t) - \nabla p \tag{2}$$

where $\mathbf{v}$ is the velocity vector, p is the isotropic pressure, and ρ is the density of fluid. The Cauchy stress tensor

$\sigma(t)$ can be resolved into:

$$\sigma(t) = \tau_N + \tau_V(t) \quad (3)$$

with τ_N a Newtonian stress tensor, corresponding to the contribution from the solvent,

$$\tau_N = 2\eta_N D \quad (4)$$

where η_N is the solvent viscosity, and D is the strain rate tensor. In Eq. (3), τ_V is the “extra” stress tensor from the contribution of viscoelastic stress. In the Oldroyd-B model, τ_V is written as:

$$\tau_V + \lambda \overset{\nabla}{\tau}_V = 2\eta_V D \quad (5)$$

where η_V is the polymeric contribution to the shear viscosity, λ is a relaxation time, and $\overset{\nabla}{\tau}_V$ is the upper convected derivatives of τ_V , i.e.:

$$\overset{\nabla}{\tau}_V = \tau_V - \tau_V \cdot \nabla v - \nabla v^T \cdot \tau_V \quad (6)$$

The elastic character of the flow is quantified by means of the dimensionless Deborah number De ,

$$De = \frac{\lambda U}{R} \quad (7)$$

where U is the velocity of the tank.

Choice of Model Parameters

Rheological characterization of the Boger fluid has been performed by Shiang *et al.*^[3] The zero-shear viscosity (η_0) is 43 Pa · s, solvent viscosity (η_N) is 28 Pa · s, and the polymeric contribution to the total viscosity (η_V) is $\eta_0 - \eta_N = 15$ Pa · s. The ratio of η_V to η_N is 0.536. The relaxation time (λ) was determined to be 1.8 s. In this work, we use the same rheological parameters as Shiang *et al.*, the fits of the Oldroyd-B model to the data for shear viscosity η_s , and the first normal stress coefficient ψ_1 are shown in Fig. 2.

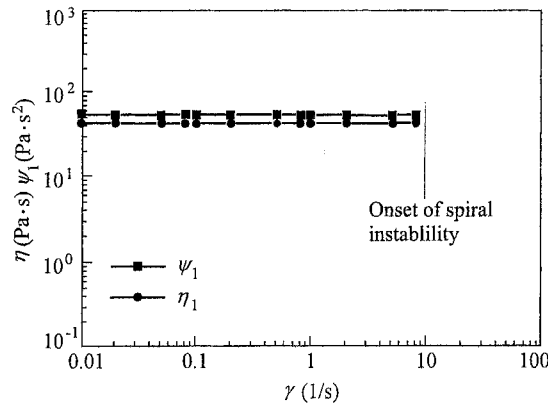


Fig. 2 Oldroyd-B predictions of shear viscosity η , first normal stress coefficient (ψ_1)

SIMULATION RESULTS

For the problem considered here, the inertial effect of the flow is omitted. No-slip physical boundaries are applied on solid walls. The radius of the cylinder R and the velocity of the tank U are used to scale all other lengths and the velocities, respectively.

The initial and the final mesh are presented in Fig. 3. Local mesh refinement is incorporated in the regions near the cylinder in the initial mesh. The time-steps for the flow at $De = 1.2$ and $De = 3.0$ are set to be 0.01 and 0.002, respectively. All calculations are performed on a DEC-Alpha 600 workstation, for the flow at $De = 3.0$,

the mean CPU time required for each Eulerian step is approximately 130 s.

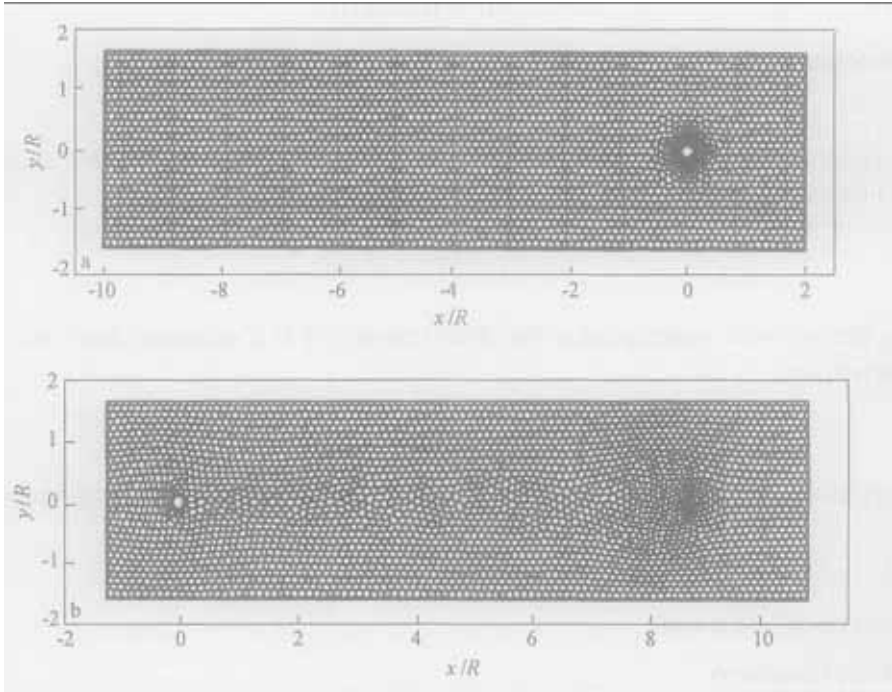


Fig. 3 Initial (a) and final (b) Voronoi meshes (3265 Voronoi cells)

The Lagrangian-Eulerian simulation proceeds by advancing the solution from start-up Newtonian flow towards steady flow after a period of time. We demonstrate in Fig. 4 the temporal development of the streamwise velocity component u along the rear stagnation line of the cylinder. For $De = 1.2$, it can be seen from Fig. 4(a) that for the time duration of $0.0 \leq t/\lambda \leq 17.5$, the velocity profiles at each instant deviate slightly along the same mean value. This clearly indicates that the steady viscoelastic flow is reached quite rapidly, and the elastic effects have not become the dominant factor that affects the flow regime. However, for $De = 3.0$, the influence of elasticity on the flow is significant, as manifested in the time-dependent behavior shown in Fig. 4(c). As time increases, the velocity profiles shift downward, and the extent of the wake region is larger than that for $De = 1.2$. For this case, the fact that the velocity profiles in the wake region have not yet reached steady state values at the end of the simulation is not important. Our purpose is to manifest the influence of elastic effects on the transient viscoelastic flow; this can be clearly observed from the present numerical results. The experimental results of Shiang *et al.* are given in Figs. 4(b) and 4(d). The present simulation with Oldroyd-B model yields results nearly identical to the experiments in predicting velocity distributions along the downstream centerline.

The time-dependent behavior for the flow at high flow rate, i.e. $De = 3.0$ is mainly caused by high transient extensional rate (du/dx) near the cylinder. According to the numerical results of velocity profiles, we calculate the extensional rate along the downstream centerline for the flows at $De = 1.2$ and 3.0 . The results corresponding to $t/\lambda = 12.0$ are plotted in Fig. 5. It appears that the extensional rate in the wake region of the cylinder for $De = 3.0$ is much higher than that for $De = 1.2$. At high extensional rates, the elasticity notably influences the flow regime, and the viscoelastic flow requires a long period from start-up motion to reach the steady state because of the nonlinear variation of extra-viscoelastic stress.

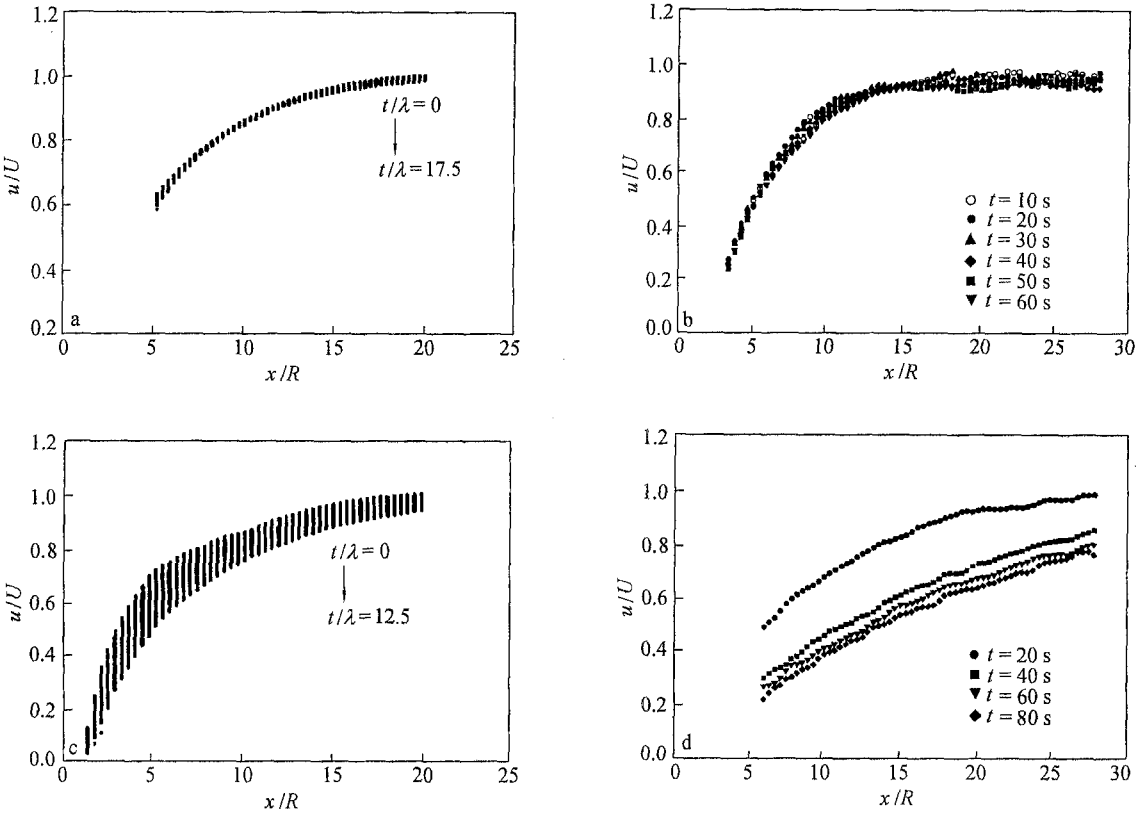


Fig. 4 Temporal development of streamwise velocity component u along the rear stagnation line of the cylinder
a) $De = 1.2$, numerical results
b) $De = 1.2$, experimental results^[3]
c) $De = 3.0$, numerical results
d) $De = 3.0$, experimental results^[3]

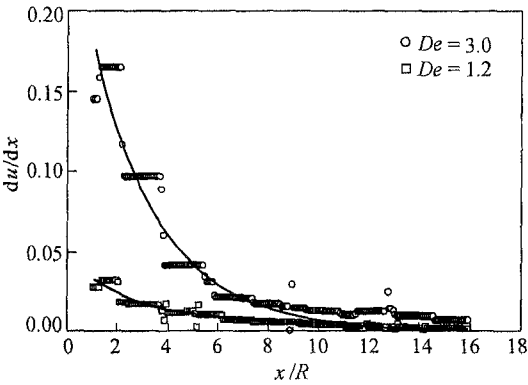


Fig. 5 Extensional rate along the downstream centerline for the flows at $De = 1.2$ and 3.0 , the results corresponding to $t/\lambda = 12.0$

To show the transient effect in detail, we concentrate our attention to the flow at high flow rate, where $De = 3.0$. In this case, the evolutions of streamwise and normal velocity components (u, v) along several baselines near the cylinder are discussed. The locations of streamwise and normal baselines in the downstream wake of the

cylinder are presented in Fig. 6. The numerical results of velocity profiles along the baselines are plotted in Fig. 7 and Fig. 8, and the experimental results of Shiang *et al.* are reiterated for comparison. It can be seen from these figures that the effect of the obstruction lasts more than $20R$ along the streamwise direction and $4R$ along the normal direction. In addition, the velocity profile shows slight asymmetry with respect to the center

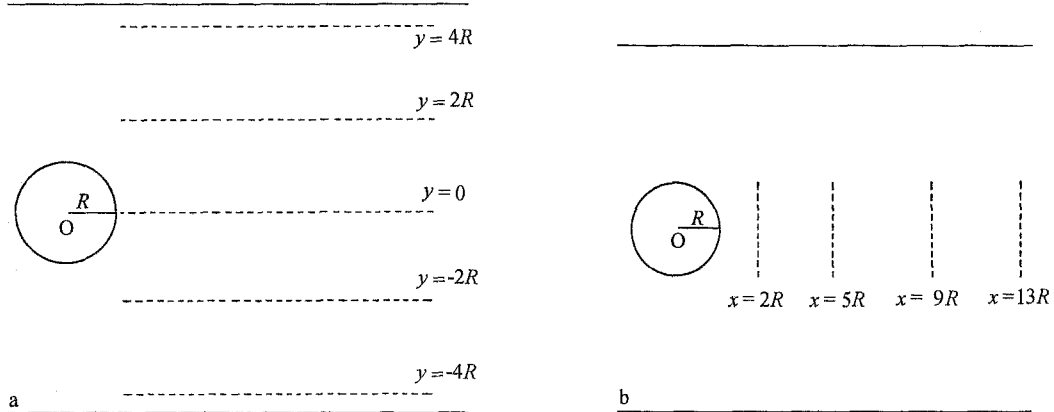


Fig. 6 Positions of streamwise and normal baselines along which the velocity profiles are calculated

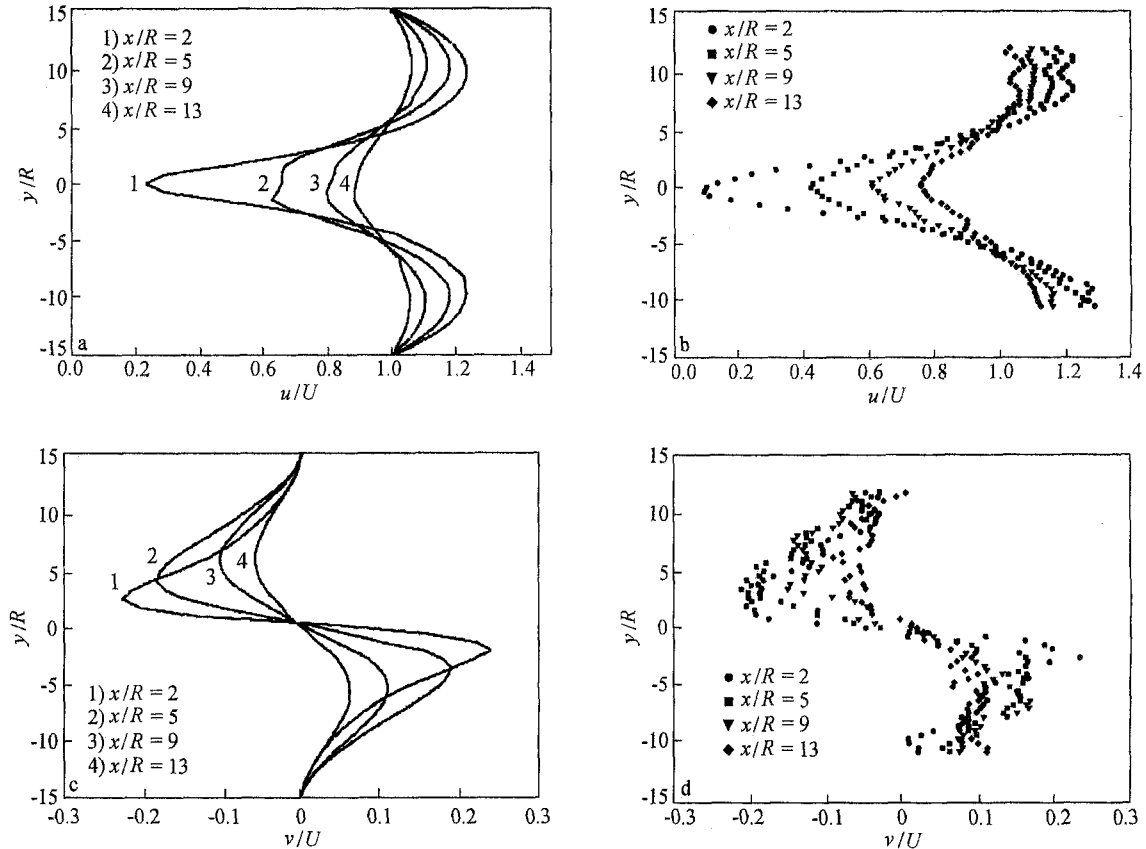


Fig. 7 Predictions of dimensionless velocity (a) streamwise component u and (c) normal component v along streamwise-sections at $De = 3.0$
The results corresponding to $t/\lambda = 8.0$, and the experimental results of Shiang *et al.* are plotted in (b)^[3] and (d)^[3] for comparison.

plane ($y = 0$). Note that at the inception of the flow, when the extra-viscoelastic stress is small, the flow is close to Newtonian, no obvious asymmetry of velocity profile has been found along symmetric positions to the center plane. As time increases, the extra-viscoelastic stress (especially the normal stress) grows, and the asymmetric flow regime appears gradually. This indicates that the observed transient effects are relevant to the elasticity of the fluid. Careful comparisons between the numerical results and the experimental observations reveal fairly good agreement for the velocity distributions along the baselines. However, from Fig. 7(a), it can be seen that the numerical results of velocity component u are slightly larger than the experimental results in the regions near the center plane.

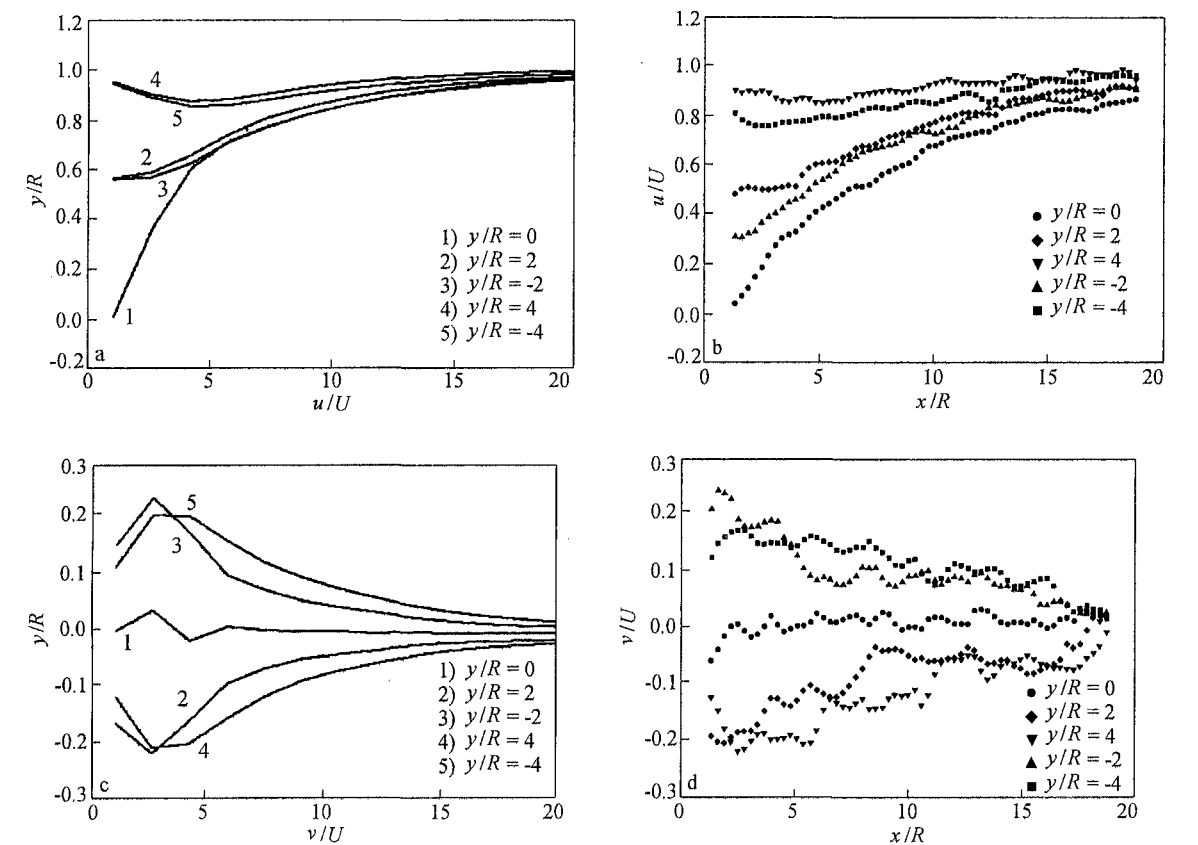


Fig. 8 Predictions of dimensionless velocity (a) streamwise component u and (c) normal component v along normal-sections at $De = 3.0$
The results corresponding to $t/\lambda = 8.0$, and the experimental results of Shiang *et al.* are plotted in (b)^[3] and (d)^[3] for comparison.

CONCLUSIONS

The aim of this work is to discuss the transient effects of a Boger flow caused by elasticity under different flow conditions. The instant velocity profiles along the downstream center plane have been obtained at different flow rates, i. e. $De = 1.2$ and 3.0 . For small De numbers, the steady viscoelastic flow is reached quite rapidly, and the elastic effects have not become the dominant factor that affects the flow regime. However, for high De numbers, the influence of elasticity on the flow is significant, and the viscoelastic flow needs quite a long period to reach the steady state. The time-dependent behavior for the flow at high flow rate is mainly caused by high transient extensional rate near the cylinder. Careful analysis of the velocity distributions along streamwise and normal baselines near the cylinder reveals that the flow regime shows slight asymmetry with respect to the center plane ($y = 0$) for $De = 3.0$. Comparisons between the results of velocity obtained by current simulation and the

experimental results given by Shiang *et al.* gave fairly good agreement.

In manifesting the influence of elastic effects on the transient viscoelastic flow, the Lagrangian-Eulerian scheme has been used in this work. Since the method is designed for time-dependent flows, it has the advantage of recording the transient behavior of fluid elements straightforwardly. Associated with the Oldroyd-B model, the method performs very well in physically reproducing the experimental observations of Shiang *et al.*

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